1 Show that $\displaystyle\int_1^2 \frac{1}{\sqrt{3x-2}}\,dx = \tfrac{2}{3}$. **[5]**

2 Fig. 9 shows the curve $y = f(x)$, which has a y-intercept at P(0, 3), a minimum point at Q(1, 2), and an asymptote $x = -1$.

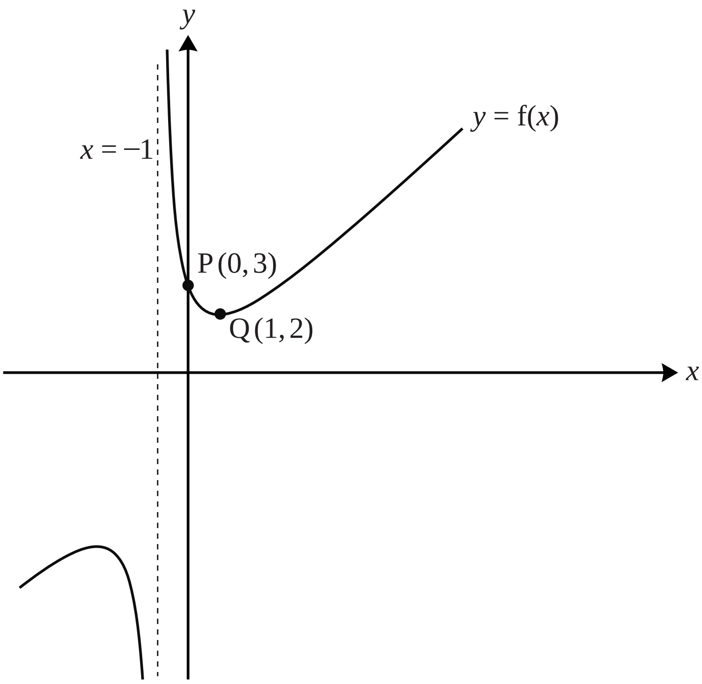

Fig. 9

(i) Find the coordinates of the images of the points P and Q when the curve $y = f(x)$ is transformed to

(A) $y = 2f(x)$,

(B) $y = f(x + 1) + 2$. **[4]**

You are now given that $f(x) = \dfrac{x^2+3}{x+1}$, $x \neq -1$.

(ii) Find $f'(x)$, and hence find the coordinates of the other turning point on the curve $y = f(x)$. **[6]**

(iii) Show that $f(x-1) = x - 2 + \dfrac{4}{x}$. **[3]**

(iv) Find $\displaystyle\int_a^b \left(x - 2 + \frac{4}{x}\right) dx$ in terms of a and b.

Hence, by choosing suitable values for a and b, find the exact area enclosed by the curve $y = f(x)$, the x-axis, the y-axis and the line $x = 1$. **[5]**

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3 Evaluate $\int_0^{\frac{1}{6}\pi} \sin 3x \, dx$. **[3]**

4 Fig. 8 shows the curve $y = f(x)$, where $f(x) = \dfrac{1}{1 + \cos x}$, for $0 \leqslant x \leqslant \frac{1}{2}\pi$.

P is the point on the curve with x-coordinate $\frac{1}{3}\pi$.

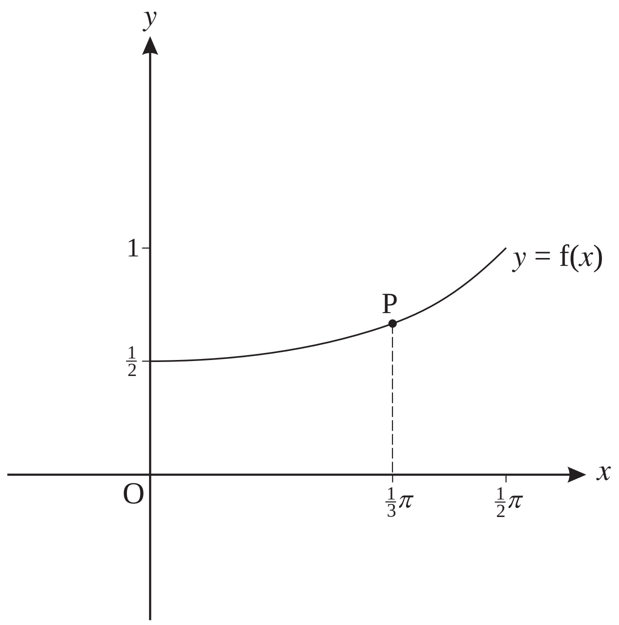

Fig. 8

(i) Find the y-coordinate of P. [1]

(ii) Find $f'(x)$. Hence find the gradient of the curve at the point P. [5]

(iii) Show that the derivative of $\dfrac{\sin x}{1 + \cos x}$ is $\dfrac{1}{1 + \cos x}$. Hence find the exact area of the region enclosed by the curve $y = f(x)$, the x-axis, the y-axis and the line $x = \frac{1}{3}\pi$. [7]

(iv) Show that $f^{-1}(x) = \arccos\left(\dfrac{1}{x} - 1\right)$. State the domain of this inverse function, and add a sketch of $y = f^{-1}(x)$ to a copy of Fig. 8. [5]

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